Proceedings of the 9ICEG
9th International Congress on Environmental Geotechnics
25-28 June, 2023 | Chania, Greece
© Authors: All rights reserved, 2023
https://doi.org/10.53243/ICEG2023-211

Assessment of Pont de Pierre settlements by Bayesian Dynamic Linear Model (BDLM) – Comparison with Hydrostatic-Season-Time (HST) model and benefits for the Pont de Pierre reinforcement works monitoring

G. Valdeyron[1], P. Losset[2], S. Dubroca[3], M. Mariko[4], S. Bonelli[5]

[1]*Geotechnical Engineer, Cerema, Bordeaux, France, email: gilles.valdeyron@cerema.fr*

[2]*Structural Engineer, Cerema, Bordeaux, France, email: patrick.losset@cerema.fr*

[3]*Structural Engineer, Cerema, Bordeaux, France, email: stephane.dubroca@cerema.fr*

[4]*Structural Engineer, Bordeaux Métropole, Bordeaux, France, email: mmariko@bordeaux-metropole.fr*

[5]*Reseacher, RECOVER, INRAE and Aix-Marseille Univ., Aix-en-Provence, France, email: stephane.bonelli@inrae.fr*

ABSTRACT

The «Pont de Pierre» of Bordeaux (France), built between 1810 and 1821 is a strategic axis that is used every day by 60,000 travelers (on foot, by bicycle or by streetcar). Since the beginning of construction, bridge piers, particularly on the left bank of the Garonne, have been affected by excessive settlements, resulting in structural damages. In 1992-1994 and 2002-2003, 16 micropiles drilled in each pier's masonry reinforced the first six piers (from the left bank to the right bank). In 2004, in order to protect the bridge piers from the impact of the Airbus barge, mooring dolphins were driven near them. However, significant additional settlements were observed. During works, settlements were carefully measured and monitoring threshold were defined (vigilance and alert thresholds). An accurate analysis of the monitoring data (before, during and after works) is undertaken here in order to separate hydraulic, seasonal and creep effects. Two different methods are used: HST method originally developed for dam engineering and a Bayesian Dynamic Linear Model commonly used for Structural Health Monitoring. The results are compared and the benefits for the micropiles reinforcement works of the last 11 piers are discussed.

Keywords: Bridge, Micropiles, *Hydrostatic Seasonal Time Model, Bayesian Dynamic Linear Model*

1 GEOTECHNICAL, HYDRAULICAL AND STRUCTURAL CONTEXT

1.1 Geotechnical and geological context

The Stone Bridge is 486 m long and composed by 2 abutments on each side and 17 low masonry arches with an average span of 23 m. All piers are founded on the Garonne River. The ascending order for piers number is from left to right river border side. From top to bottom the ground foundation is composed by recent alluvial deposits (silty clays or silty sands) underneath, a layer of coarse sand and gravel is encountered which is underlying marls and limestone substratum. The upper layer is getting less and less clayey and thinner from left to right border of the Garonne. Due to poor mechanical properties of the upper layer of the ground foundation, each pier and abutments are founded on approximatively 250 wooden piles anchored in the sand and gravel layer. On the left border of the Garonne (that concerns the left abutment and the piers 1 to 4), driven wooden piles were setted up with a lower energy than the others, it entails a weaker anchorage of piles.

1.2 Hydrological context

The bridge is subjected the Garonne water level variations that are mainly influenced by tidal cycles. Each cycle is about 12:25 hours and consists in two high and two low tides per day. This implies a cyclic action on the wooden piles foundations, due to flotation, of about 5 m, or which is equivalent, a 10% variation of the load on each pier and abutments foundation. Indeed, at high tide, due to buoyancy, permanent loads on foundations are reduced – rather, the loading of the foundations is maximum at low

tide. In addition, the bridge is affected by scours on piers sides but also in front of and beside the bridge where sub-river slopes are drawing up to the foundations. To ensure the stability of the sub-river slopes, rip rap works were carried out in 2016-2018.

1.3 Structural context

Aware of the difficulties ensure a perennial foundations for the bridge, designers have constructed a system of inside arches in each pier in order to reduce their weight (see Figure 1). Theoretically, it provides to the Pont de Pierre a complex behaviour between an arch bridge and a box girder bridge. Moreover, a tramway platform, made of a concrete slab, have been added on the extrados bridge; that emphasis previous consideration.

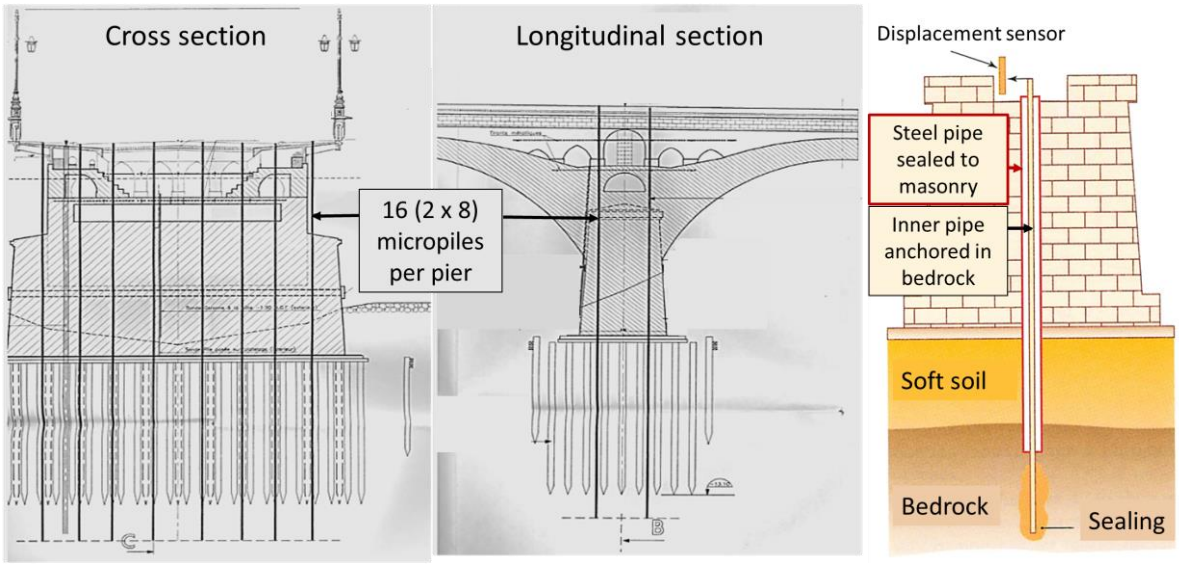


Figure 1. *Principle of micropiles reinforcement in transversal (left) and longitudinal section (center) and principle of fixed point (right)*

Since the beginning of construction the bridge has been affected by excessive settlements, as explained by a weaker anchorage of piles, the left border was the first to be affected (left abutment and piers 1 to 4). The first foundations reinforcements have been realized in 1993-1994 on piers 1 to 4 by sealing to the masonry and marls a set of 16 micropiles per pier (2 lines of 8 micropiles). The aim was to provide additive resistance to the wooden piles foundations in order to limit settlements. Then in 2002-2003 piers 5 and 6 were treated with the same process. The design of these reinforcement are illustrated in Figure 1. Nowadays, studies have started to design the reinforcement of piles 7 to 16 and abutments. In this context, geotechnical investigations have been carried out in April-May 2021 throughout piers and foundations of piers 7 to 16 and abutments and two micropiles test have been bored and sealed on pile 8 in October 2022. In order to protect the bridge against Airbus barge impact, dophins were driven in the vicinity of piers 8 and 9 in 2004.

Since 1992, settlements are measured by a fixed point system installed on each piers and consisting on a micropile bored throughout the pier, anchored (about 6 m) in the marls (fixed point), and with a free length all along the alluvium and the masonry (Bru, 1994). In the alluvium the free movement is provided by a larger diameter pipe that surrounds the micropile as illustrated is Figure 1. The monitoring system is composed by 18 (one on each pier and abutment) highly precise inductive displacement transducers (LVDT), in addition, 18 clinometers are installed to measure the upstream/downstream inclination. By default, the time step is 1 measurement per hour, nevertheless during works on the bridge, this time step was decrease to 4 measurements per hour in order to enhance analyses of data monitoring and relationships with works. The accuracy of the settlement gauges is 1 µm. There is no direct level measurement of tide level on the bridge, tide levels are measured by tide gauges located 4 km downstream from the bridge in the Bordeaux harbour. Some underpinned micropiles (4 of 16) of piles 1 to 6 were monitored by stress gauges in order to measure the load transfer process from wooden pile foundation to the micropiles.

2 THE HYDROSTATIC-SEASON-TIME (HST) MODEL

The tools most commonly used for dam monitoring data analysis purposes are the statistical methods of the Hydrostatic-Season-Time (HST) type. These methods were first developed in the 1960s to analyze the displacements resulting from the pendulum effects occurring at arch dams (Ferry and Willm, 1958). They are still being used in several countries to analyse measurements of other kinds (Guedes and Coehlo 1985, Carrère et al. 2000, Bonelli 2008 and 2009). The HST model is based on effects of three kinds. First, there are the hydrostatic effects, which correspond to the variations H resulting from changes in the water level. This variable is given by a polynomial of order 4 (parameters a_1, a_2, a_3, a_4):

$$H(t) = a_1 z(t) + a_2 z^2(t)) + a_3 z^3(t) + a_4 z^4(t) \qquad (1)$$

With $z(t) = (Z(t) - Z_{min})/(Z_{max} - Z_{min})$, where $Z(t)$ is the reservoir level (minimum value Z_{min}, maximum value Z_{max}).

The second term is the time of year, which accounts for seasonal variations S in the measurements during twelve-month and six-month periods, is described by the following expression (parameters A_1, d_1, A_2, d_2):

$$S(t) = A_1 sin(\omega_a(t - d_1)) + A_2 sin(2\omega_a(t - d_2)) \quad (2)$$

where $\omega_a = 2\pi/T_a$ is the annual angular frequency (T_a corresponds to a one-year period).

The seasonal variations effect, assumed to be reversible, is quantified by maximum amplitudes A_i and lag time d_i. One of the oldest known application of this model was developed by Forbes (1846) who used the so called “sinusoidal adjustment method” to account for cyclic variations in the soil temperature.

The third term accounts for the time-dependent trends in the ageing processes. This variable is often called the “irreversible effect”. It is expressed as follows (parameters c_1, c_2, c_3):

$$T(t) = c_1\tau + c_2 exp(\tau) + c_3 exp(-\tau) \qquad (3)$$

with $\tau = (t - t_0)/\Delta T$, where t is time, and $[t_0, t_0 + \Delta T]$ is the analysis interval. The settlement rate is given by the time derivative, estimated in the middle of the analysis interval:

$$v_1 = [c_1 + c_2 exp(1/2) - c_3 exp(-1/2)]/\Delta T \qquad (4)$$

The HST model $Y(t) = Y_0 + H(t) + S(t) + T(t)$ has 11 parameters estimated by multiple linear regression with MSE $||y - Y||^2$ where y is the measurement. This approach has been classically used to analyze dam monitoring data. The experience acquired at several hundreds of dams has confirmed what an excellent tool this approach can be for interpreting monitoring data. It has also been used in many other fields (Young, 1998). In the present work, the HST model is applied to vertical displacement analysis measured on the Pont de Pierre.

3 THE BAYESIAN DYNAMIC LINEAR MODEL (BDLM)

Initially and mainly used in econometrics, BDLM have been recently developed for Structural Health Monitoring issues. In this paper, all analyses have been carried out with the practical open-source software for time series analysis called OpenBDLM (Gaudot, 2019). Theoretical aspects are extensively exposed in the bibliography (Goulet, 2017 and 2018) and can be briefly summarized as follows:

Observations $\boldsymbol{y}_t$ (for example settlement measurements) are described, for each time t between 1 and T, by a superposition of hidden states $\boldsymbol{x}_t$:

$$\boldsymbol{y}_t = \boldsymbol{C}_t\boldsymbol{x}_t + \boldsymbol{v}_t, \begin{cases} \boldsymbol{y}_t \sim \mathcal{N}(\mathbb{E}[\boldsymbol{y}_t], cov[\boldsymbol{y}_t]) \\ \boldsymbol{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t) \\ \boldsymbol{v}_t \sim \mathcal{N}(0, \boldsymbol{R}_t) \end{cases} \qquad (5)$$

Where $\boldsymbol{C}_t$ is an observation matrix which indicate which hidden state contributes to the observations and $\boldsymbol{v}_t$ is the observations error. Each quantity is supposed to be a Gaussian random variables. A transition model permits to associate the variation of the hidden states at time t knowing his value between 1 and t-1, this transition model is describe by a matrix $\boldsymbol{A}_t$. A transition model error $\boldsymbol{Q}_t$ is also associated to $\boldsymbol{A}_t$:

$$\boldsymbol{x}_t = \boldsymbol{A}_t \boldsymbol{x}_{t-1} + \boldsymbol{w}_t,\ \boldsymbol{w}_t \sim \mathcal{N}(0, \boldsymbol{Q}_t) \qquad (6)$$

A BDLM is built by representative blocks of different possible behaviours, generally $\boldsymbol{x}_t$ can be handled with a combination of:

- Local Level (LL): baseline response of the structure;
- Local Trend or Local Acceleration (LT or LA): linear or accelerated response of the structure for example ageing component of the structure;
- Periodic: harmonic component such as seasonal effects;
- Auto-Regressive component: time-dependent model prediction errors (what is not caught by previous components).

For example, considering the Local Level block:

$$\boldsymbol{x}^{LL} = x^{LL}, \boldsymbol{A}^{LL} = 1, \boldsymbol{C}^{LL} = 1, \boldsymbol{Q}^{LL} = (\sigma^{LL})^2 \qquad (7)$$

Where σ^{LL} is the model error term.

The aim of BDLM is to forecast the structure response at time *t+n* ($\boldsymbol{y}_{t+n}$) beyond the current time *t*. To do that a Kalman filter algorithm, divided in two parts (prediction step and measurement step) is used. In the prediction step the probability hidden state values $\boldsymbol{x}_t$ a time t, is calculated on the basis of a set of known observations $\boldsymbol{y}_{t-1}$ between 1 and *t*-1. In the measurement step, $\boldsymbol{y}_t$ is used in order to correct the estimation previously given. Then optimization of the model parameters is given by a technique of Maximum Likelihood Estimation which seeks the maximisation of the logarithm of the prior probability of the observation $\boldsymbol{y}_t$ at time *t* knowing the state estimated at time *t*-1 (prediction step).

4 APPLICATION TO THE PONT DE PIERRE AND RESULTS

4.1 Results of HST Model

HST Model was developed for dam engineering where the variable H is resulting on level water variation of the reservoir. In our case, to take into account that high water level has a positive effect on settlements (due to buoyancy as previously explained), the H variable is modified by the way of $z(t)$ to:

$$z(t) = \left[\left(G(t) - W(t)\right)_{max} - \left(G(t) - W(t)\right)\right] / \left[\left(G(t) - W(t)\right)_{max} - \left(G(t) - W(t)\right)_{min}\right] \quad (8)$$

Where $G(t)$ is the permanent load of the pier and $W(t)$ is the buoyancy force due to tide water level variations. Thus, $z(t)$ is the loading ratio of the foundation, it equals 1 at minimum measured low tide level and 0 at maximum measured high tide.

For each pier (P01 to P16) and abutment (left border CRG and right border CRD), the analysis has been carried out according different time intervals:

- Before the reinforcement works of piers P05 and P06 by micropile, between 1992 to 2002 ;
- After reinforcement works of P05 and P06, between 2003 to 2021, i.e. 19 years (approximatively 145,000 measurements).

4.1.1 Analysis of settlement measurements before reinforcement

Before reinforcements, overall results are exposed for P05, P06 and P09 (see Figures 2 to 4). The HST Model is able to separate different contributions on settlements as: seasonal influence (outdoor temperature effect), water level influence (tide effect through buoyancy) and ageing of the structure (creep). Before reinforcement, the model fitting is quite accurate. For all piers, except P05, irreversible effects are mainly fitted with a linear creep function of time. Globally, the HST model is able to explain more than 99 % of displacement measurements. Effect of ageing is representing about 99% of total displacement, seasonal effect represents only 0.1% and tide effect is evaluated as zero.

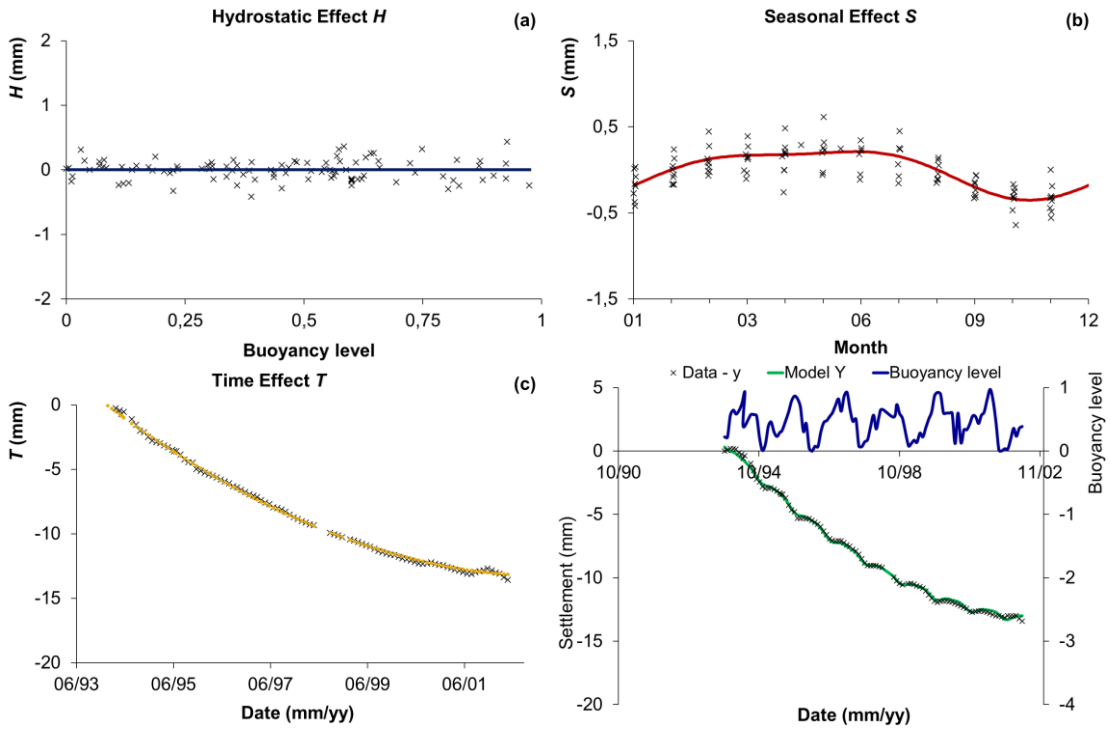


Figure 2. *Results of HST analysis of P05 measurements on 1992-2002 (before reinforcement): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

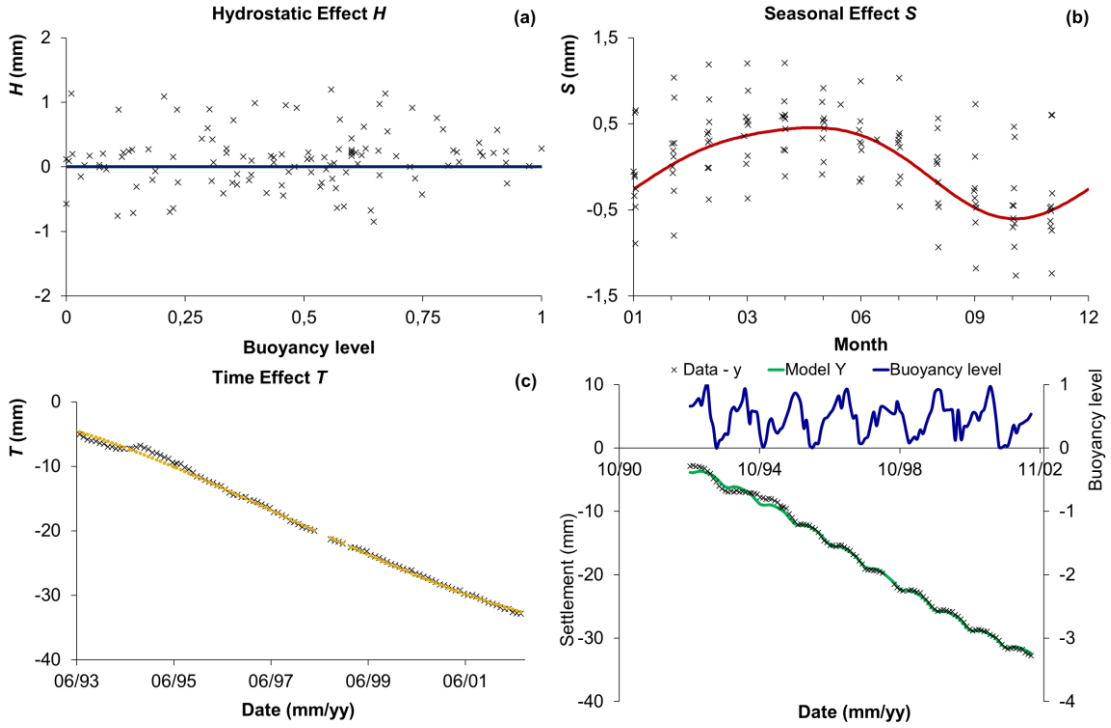


Figure 3. *Results of HST analysis of P06 measurements on 1992-2002 (before reinforcement): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

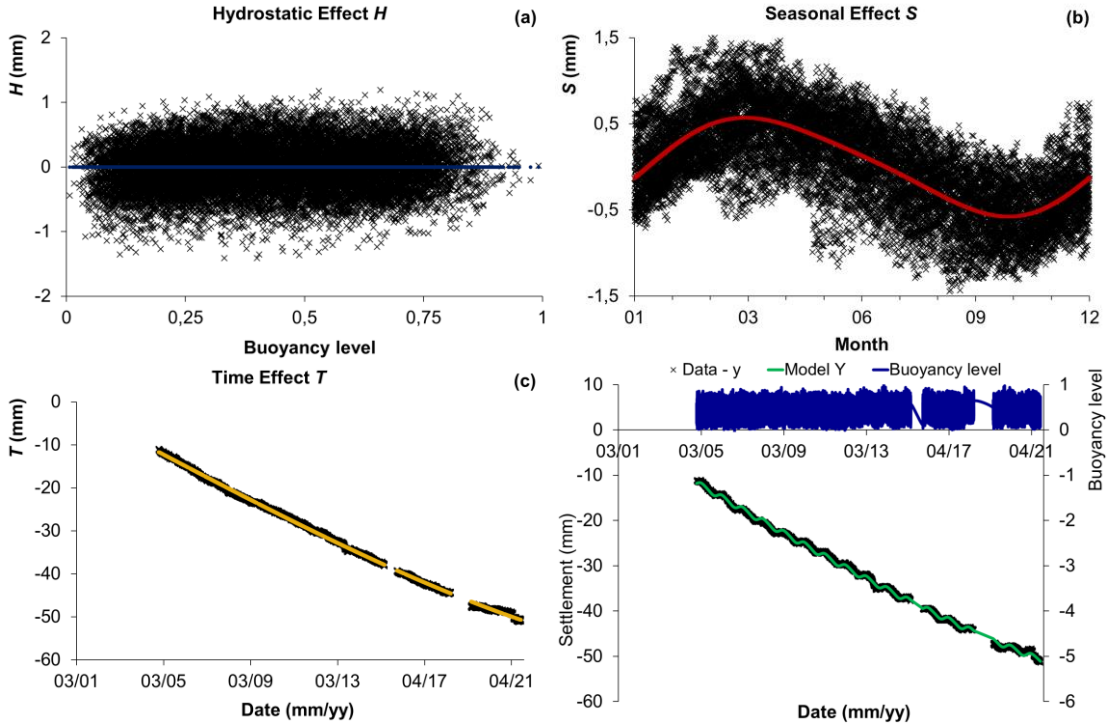


Figure 4. *Results of HST analysis of P09 measurements on 2005-2021 (unreinforced pier): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

4.1.2 Analysis of settlement measurements after reinforcement

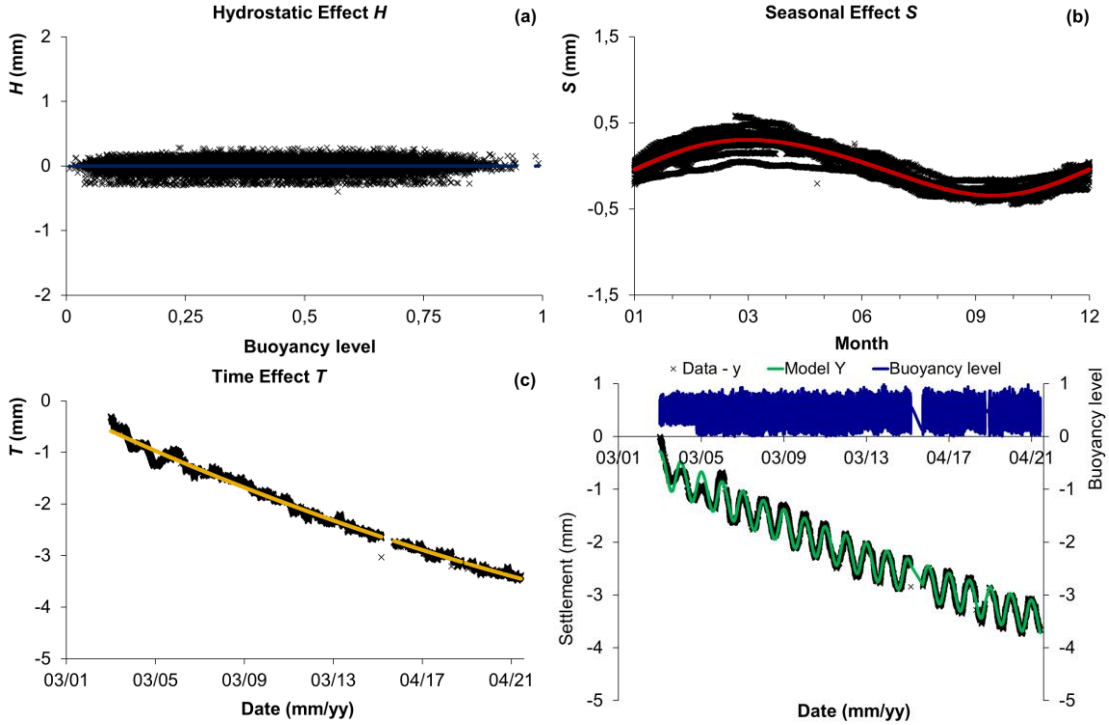


Figure 5. *Results of HST analysis of P05 measurements on 2003-2021 (after reinforcement): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

After reinforcement, settlements are hugely damped (see Figure 5 and 6) that illustrates the benefits of micropiles reinforcement and the ability of the model to handle effect of reinforcement on structure behaviour, that essentially consists to improve the stiffness of the global foundation of concerned piers.

Seasonal effects, considered as reversible effects, are fitted with a twelve months period sinusoidal function, all piers are synchronous and abutments are in opposite phase. Amplitudes of variations are all between 0.3 and 0.5 mm. Amplitude of seasonal effect are unchanged by reinforcement, which confirms that this contribution only includes effects of the temperature and not the summer low-flow of the Garonne (in this case, due to buoyancy, the load on the foundation is at his peak value, and settlements could have been maximized). Indeed, if summer low-flow was included in the seasonal terms, the amplitude of seasonal effects would have been reduced by reinforcement (as it enhances the foundation stiffness), and this not the case. Concerning water level effect (i.e. tide effect), the mean value of settlements is zero and the amplitude of measurements is reduced in comparison with non-reinforced case, this is consistent with the increased stiffness of foundation.

Concerning the tide effects on displacement, it is shown that this contribution is next to nothing and settlements due to water level variation (buoyancy) are reversible.

Then it appears that HST model can explain 99% of displacement, amongst them, 85 to 90% are due to the ageing of the structure. Seasonal effects represents only 7 to 15% of total displacement and reversible tide effects are less than 0.1%. It also significate that the model doesn't need further information to explain settlements of the bridge, particularly tramway traffic contribution to settlements is insignificant.

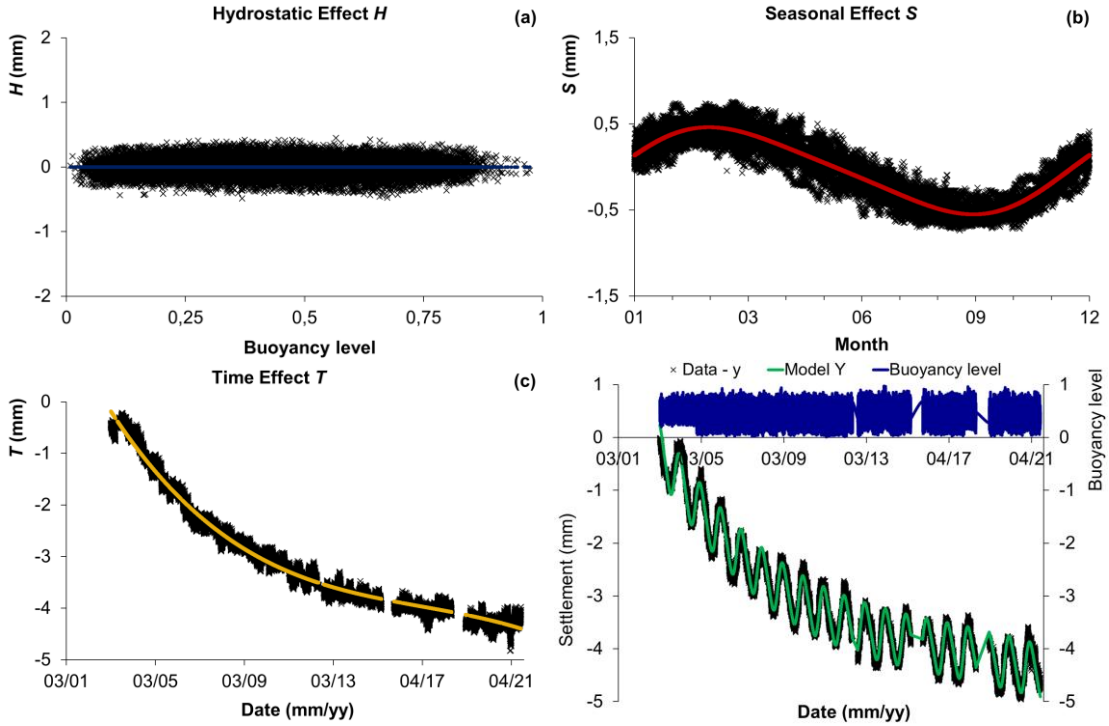


Figure 6. *Results of HST analysis of P06 measurements on 2003-2021 (after reinforcement): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

4.1.3 Thermal behaviour of the bridge

For piers, previously exposed results shows that maximum settlements are measured during warm season, this result is consistent with dam data monitoring analysis of inverted pendulums. During summer, temperature rise entails a lengthening of arches between piers and thus an increase of settlements – this behaviour is consistent with arch bridges. For a box girder bridge model with fixed point on the basis of the foundation, the behaviour is opposite: during summer, temperature rise and associated gradient entails a lengthening of the piers and, as a consequence, a minimum settlement would be obtained during warm season. In addition, when the number of span is increasing, thermal effects are only limited to edge effects (see Figure 7d).

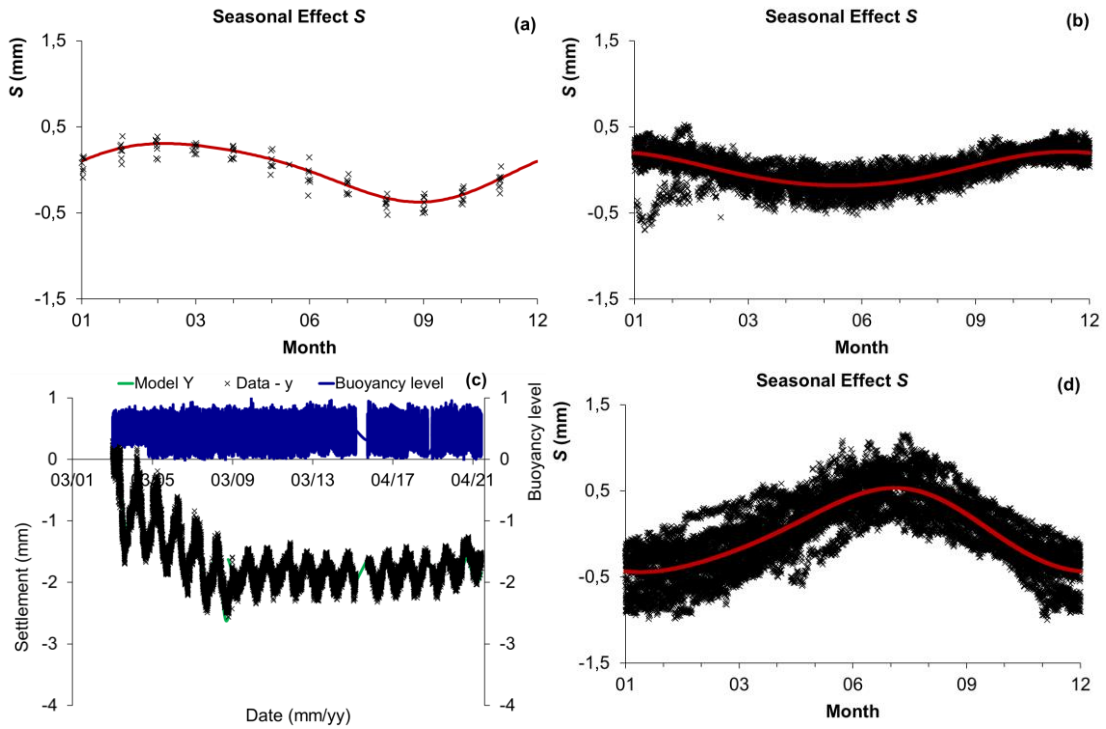


Figure 7. *Evolution of seasonal effects: results of HST analysis of P01 (after reinforcement) and CRG: (a) P1 1995-2002, (b) P1 2009-2021, (c) P1 Data, Model, Buoyancy level, (d) CRG (2003-2021)*

The effect of reinforcement on the behaviour of P01 is highlighted according three periods analysis (1995-2002, 2006-2008 and 2009-2021). Results obtained by HST are remarkable (see Figure 7): before the absolute stabilisation of settlements (1995 to 2008), the pier acts as other arches – and from 2009 to 2021, settlements totally stops and a part of the Pont de Pierre, between CRG to P01 acts as a box girder bridge (fixed point).

4.1.4 Analysis of micropiles stress gauges

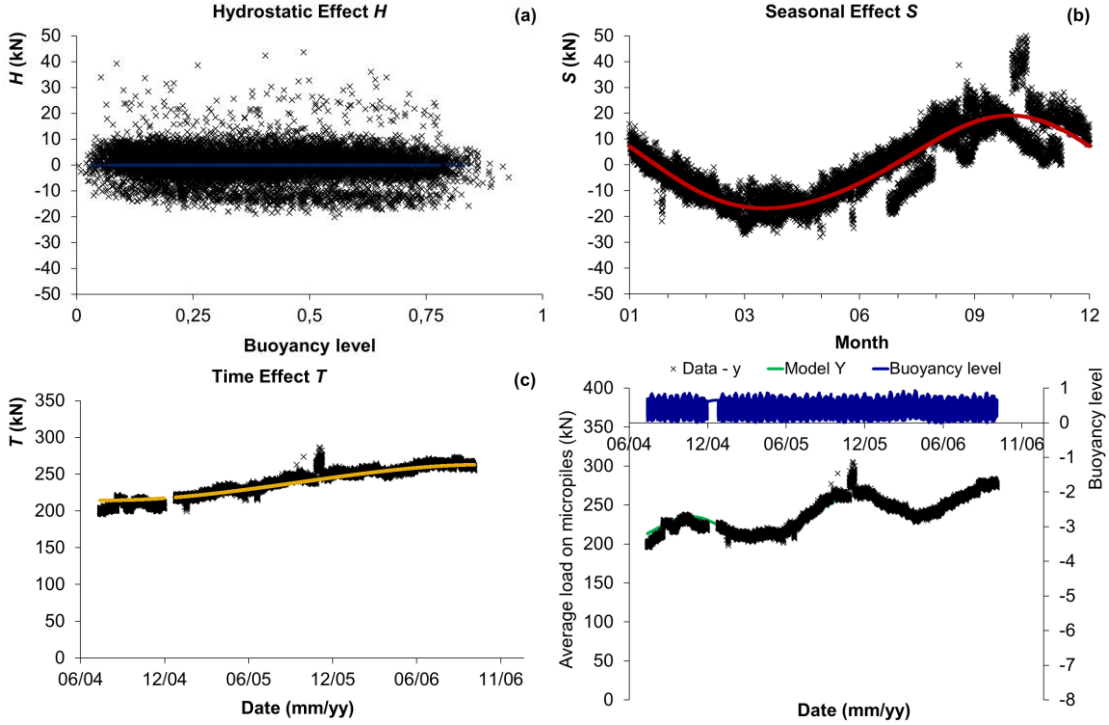


Figure 8. *Results of HST analysis of P01 measurements on 2004-2006 (after reinforcement): (a) Hydrostatic effect H, (b) Seasonal effect S, (c) Time effect T, (d) Data, Model and Buoyancy level*

The method is also applied to the stress gauges located in micropiles, results for P01 to P06 are consistent with settlement analysis as seasonal effects are in opposite phase: load transfer is maximized in warm period when settlements are also at their peak values (see Figure 8). Micropile stiffness can also be checked as the ratio of the measured force by the measured settlements. For P01, based on the HST analysis, the stiffness of each micropile is evaluated at 93 MN/m for a calculated value of 95 MN/m (considering real section and free length of pipe).

4.1.5 Global analysis of the structure

All HST analyses on the 18 instruments are summarized in Figure 9. Except for P01, amplitudes of seasonal effects (outdoor temperature) and their phase shifts are not affected by the micropiles reinforcement, for unreinforced piers, differences over the time series are attributed to the quality of the data (much more data for the 2003-2021 period) (a). Behaviour of P01 and abutments are consistent with edge effects of a box girder bridge (b). Efficiency of reinforcement of piers P01 to P06 is demonstrated regarding the estimated settlement rate (c), over the analysed intervals time, acceleration of settlement is observed for P08 to P10 and may be explained by damages due to dolphins works (2004). As remarkable, before reinforcement, creep settlements were representing about 60 to 80% of the total settlements (including seasonal effects), after reinforcement this ratio is reduced to less than 20% (knowing that settlements are stabilized with a velocity $v_1 \leq 0{,}2$ mm/year).

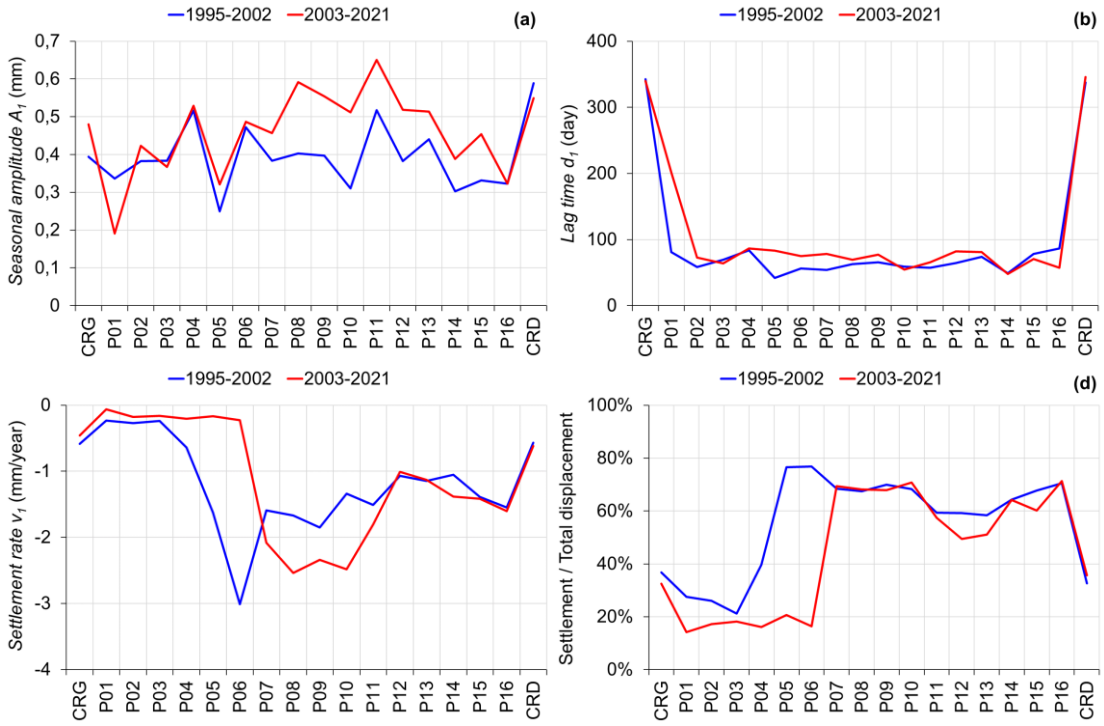


Figure 9. *Results of HST analysis for the whole bridge on two intervals (before and after reinforcement): (a) Amplitude of seasonal reversible effects A_1, (b) Lag time d_1, (c) Settlement rate v_1, (d) Settlement / Total displacement ratio.*

4.2 Results of BDLM Model

4.2.1 Comparison with HST model

BDLM analyses are carried out on P06 settlement measurements on the 2003-2021 period, with a 6 hours step time data, the model is based on 5 components: a Local Acceleration (LA) to handle the damping of the measured displacement, 3 periodic components to handle the seasonal effects (365.2422 days cycle, 182.2211 days cycle) and the tidal effect (1 day cycle), and an Auto-Regressive component (AR) to calculated what is not catched by the model. The learning period is set to 365.2422 days in order to include the seasonal period. Results are exposed on Figure 10, they are comparable to HST (respectively) in terms of seasonal amplitude: 0.48 mm (respectively 0.49 mm) for the annual cycle, 0.06 mm (respectively 0.07 mm) for the 6-months cycle. The tide effect (settlements due to buoyancy)

are calculated to 0.01 mm, less than the real measured value (0.1 mm), due to the data signal filtering (1 measurement per 6 hours) and the real tide daily period (0.518 day and not 1 day). In terms of local level, results are consistent with the estimated Time effect by HST. Beyond learning period, the Auto-Regressive component is fixed to zero, which confirms the ability of the model to analyse the given data.

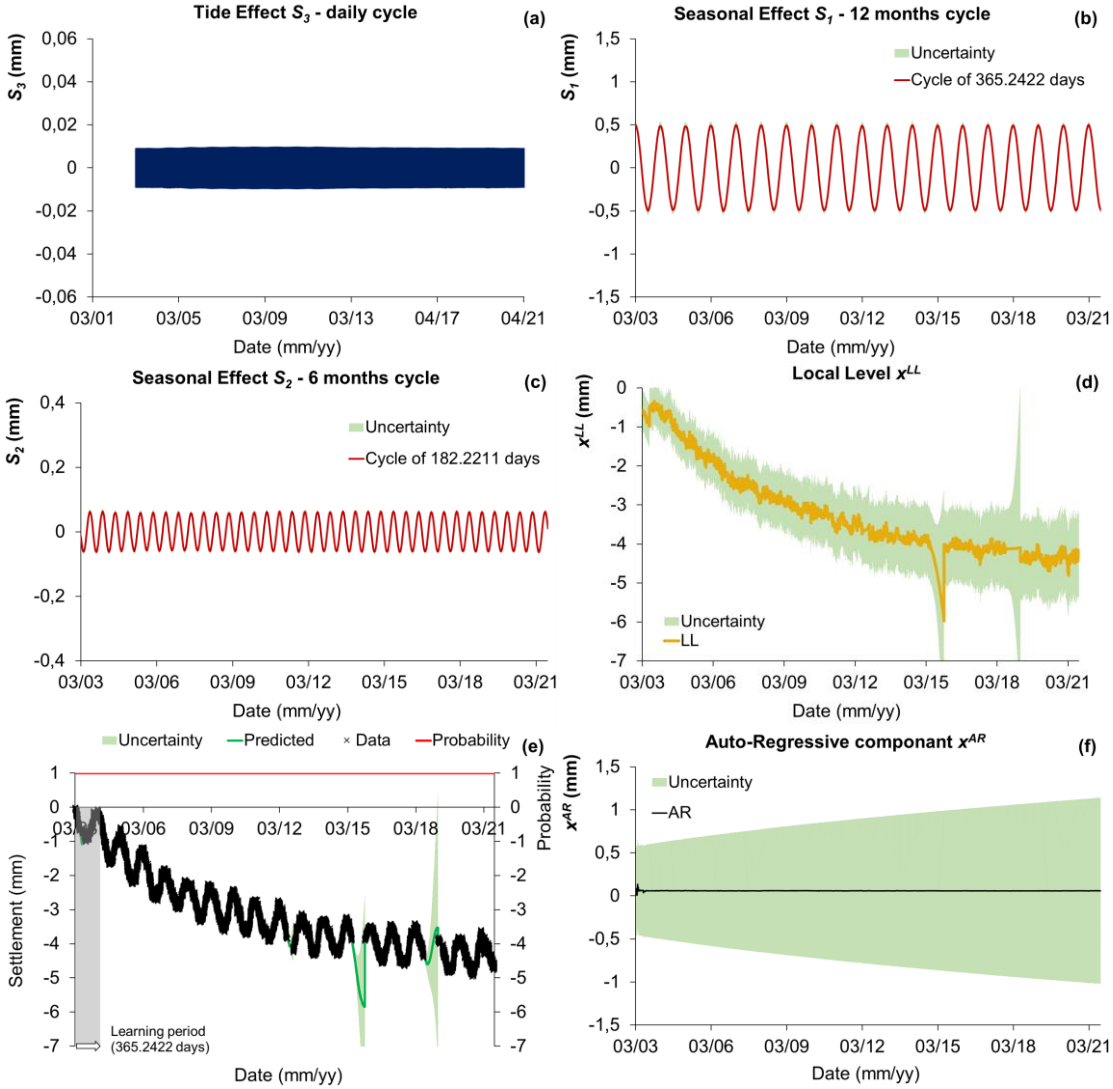


Figure 10. *Results of OpenBDLM analysis of P06 measurements on 2003-2021 (after reinforcement): (a) Daily cycle S_3, (b) 364.2422 days cycle S_1, (c) 182.2211 days cycle S_2, (d) Local Level x^{LL}, (e) Data, Model, Probability model, (f) Auto-Regressive component x^{AR}.*

4.2.2 Application to the real time monitoring works

BDLM is then used to analyse monitoring data during works and detect anomalies in the behaviour of the structure (when HST has difficulties to detect evolutions in a very short period). To do that, two models are used: the first model M_1 is fitted to the normal behaviour of the structure with a 365.2422 days learning period, and the second model M_2 is representative of the abnormal behaviour. Calculations are made on P08 and P09, with 5 components for each model: the first model is composed by a Local Trend (LT) to fit with the linear creep, 3 periodic components (same as previously exposed for P06) and an Auto-Regressive – the second model shares all components with M_1, except the LT which is substituted by a LA component. Results are reported in the Figures 11 and 12.

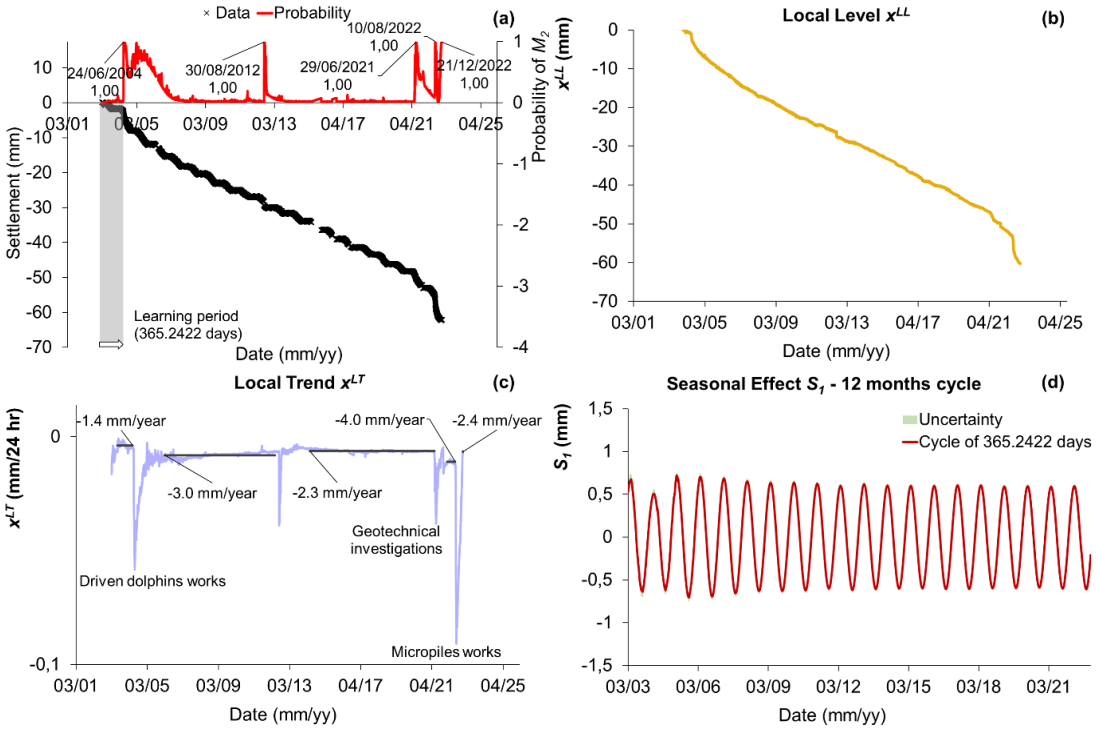


Figure 11. *Results of OpenBDLM analysis of P08 measurements on 2003-2022: (a) Data and Probability of abnormal Model M_2, (b) Local Level x^{LL}, (c) Local Trend x^{LT}, (d) 364.2422 days cycle S_1.*

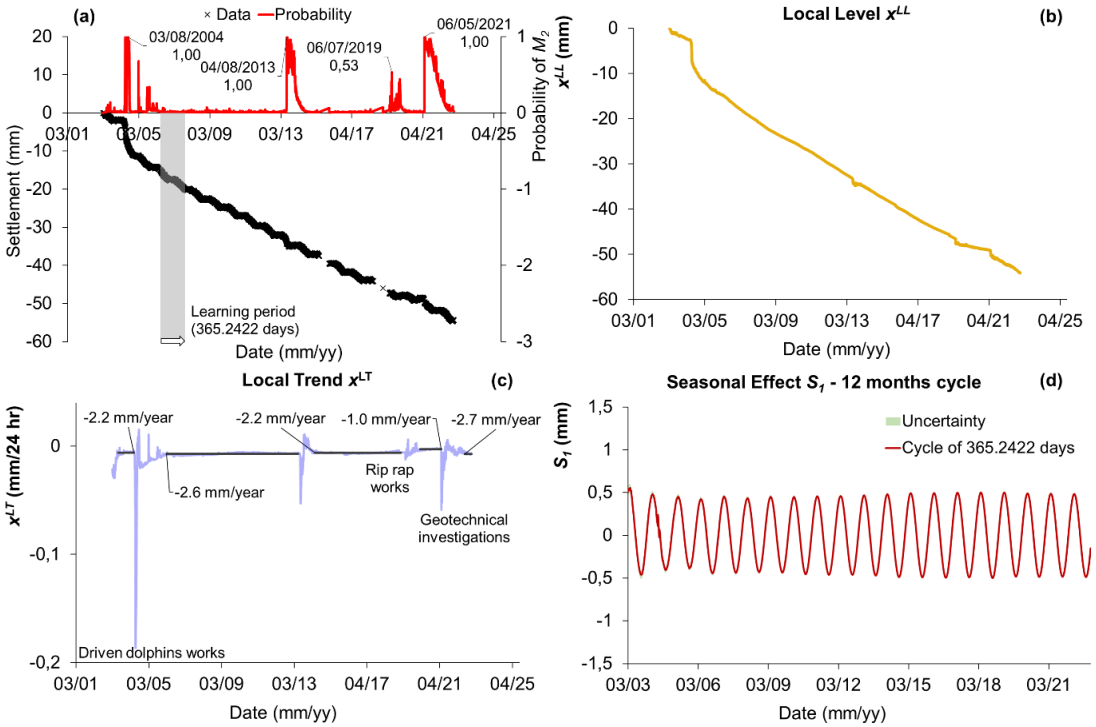


Figure 12. *Results of OpenBDLM analysis of P09 measurements on 2003-2022: (a) Data and Probability of abnormal Model M_2, (b) Local Level x^{LL}, (c) Local Trend x^{LT}, (d) 364.2422 days cycle S_1.*

For P08 and P09 both, initial velocity (v_1 = 1,4 mm/year, Figure 11a and v_1 = 2,2 mm/year, Figure 12a) is consistent with the HST results on the previous period analysis (1992-2002, v_1 = -1,6 mm/year and v_1 = -1,9 mm/year), that is convincing regarding the reduced learning period. Airbus barge protection works (driven dolphins works) are also detected (probability of abnormal model equals 1, see Figures 11a and 12a), and their impact on displacement is evaluated with an acceleration of settlements to respectively 3.0 and 2.6 mm/year. Anomalies detected in 2012 and 2013 are not explained by external reason (works or changes in bridge exploitation) and are attributed to a sensor anomaly. Especially for P09, a positive

impact on the reduction of velocity is detected in 2019, which could be explained by the sub-river slopes works stabilisation by gabions (2016-2018). In May and June 2021, geotechnical investigations realized in ground foundation of piers (pressumeter tests and core drilling) are also detected, impact on settlement rate is significative (increase from 2.3 mm/year to 4.0 mm/year for P08 and 1 mm/year to 2.7 mm/year for P09). And finally, consequences of the two micropiles tests executed throughout the masonry and wooden piles foundation of P08 are accurately handled in terms of settlement acceleration during drilling (masonry and ground foundation) and stabilization (2.4 mm/year) after sealing to the masonry. Seasonal effects due to outdoor temperature (annual cycle) are also reported, as seeable calculated amplitudes are comparable to these obtained with HST model (Figure 9a, 11d, 12d).

5 CONCLUSIONS

Sensitive structures such as The Pont de Pierre require appropriate tools to analyse monitoring data and provide convincing explanations of the structure's behaviour. Both methods explored (HST and BDLM) provide consistent results that quantify the importance of the external effects (outdoor temperature and daily tidal cycle) on the measured settlements. Applied on time series including works, BDLM identifies abnormal behaviour of the structure. This method could be advantageously used in real-time monitoring. The physical meaning of the chosen variables gives the HST method an important robustness for data monitoring and explanations of the structure behaviour. Both method should be used favourably together, one to explain structure behaviour regarding 50 years of use and validation on thousands dams (HST), the other to detect on real-time an abnormal behaviour of the structure, for instance during works (BDLM).

REFERENCES

Bonelli S. 2008. On pore-pressure analysis in earthdams. International Water Power and Dam Construc-tion 60(12):36-39.

Bonelli S. 2009. Approximate solution to the diffusion equation and its application to seepage-related problems. Applied Mathematical Modelling 33(1):110-126.

Bru, J. P. (1994). Réalisation de vrais points fixes pour la mesure des tassements des ouvrages en maçonnerie dans les zones soumises au marnage. Bulletin de Liaison des Laboratoires des Ponts et Chaussées,190, 102-106.

Carrère A., Colson M., Goguel B., Noret C. 2000. Modelling: a means of assisting interpretation of readings. Proc. XXth International Congress on Large Dams, Beijing, ICOLD, p. 1005-1037.

Ferry S., Willm G. 1958 Méthodes d'analyse et de surveillance des déplacements observés par le moyen de pendules dans les barrages. Proc.VIth International Congress on Large Dams, New-York, ICOLD, p. 1179-1200.

Forbes J.D. 1846. Account of some experiments on the temperature of the earth at different depths and in different soils near Edinburgh. Transactions of The Royal Society of Edinburgh 16:189-236.

Gaudot, I., Nguyen, L.H., Khazaeli S. and Goulet, J.-A. (2019). OpenBDLM, an Open-Source Software for Structural Health Monitoring using Bayesian Dynamic Linear Models. In the proceedings from the 13th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP13), May 2019.

Goulet, J.-A. (2017). Bayesian Dynamic Linear Model for Structural Health Monitoring. Structural Control and Health Monitoring, 24.

Goulet, J.-A, Koo, K. (2018). Empirical validation of Bayesian Dynamic Linear Model in the context of Structural Health Monitoring. Journal of Bridge Monitoring, 23.

Guedes Q.M., Coelho P.S.M. 1985. Statistical behaviour model of dams. Proc. XVth International Congress on Large Dams, Lausanne, ICOLD, p. 319-334.

Young P. 1998. Data-based mechanistic modelling of environmental, ecological, economic and engineering systems. Environmental Modelling & Software 13:105-122.